Characteristic Function of impulse responses in churches

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ABSTRACT

The tool presented enables the sound behaviour of a room to be objectified on the basis of the representation of the characteristic function (CF) of said room.

In this paper, we study characteristic functions, defined from the impulsive responses measured at different points in the room, that are obtained in churches and cathedrals. These CFs show a variation that correlates with the distance from the point of measurement and the source. An analysis is carried out on the relationship between this variation with those acoustic parameters that depend on the distance to the source: C_{50}, C_{80}, and G. In addition, the differentiating features between these types of spaces are shown, thereby allowing the characteristic function to offer an overall assessment of the room, which in turn enables the classification of similar sound spaces.

Keywords: Impulse response, characteristic function
I-INCE Classification of Subject Number: 76
http://www.upv.es/contenidos/ACUSVIRT/

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1. INTRODUCTION

The Characteristic Function of a room (CF) [1,2], is built for each source-receiver combination as a polynomial whose coefficients are the values of the squared Impulse Response (IR) from where it starts (onset) until where it ends (N). This polynomial is evaluated in M values (we have considered M = 128) that correspond to a distribution in the interval distributed as $\frac{m}{M}$, and is defined by the following expression, which is interpreted as an energy distribution taking the decimal logarithm:

$$CF(m) = 10 \cdot \mathrm{Log}_{10}\left(\sum_{n=1}^{N} h^2(n) \cdot \left(\frac{m}{M}\right)^n\right) m \in [1, M] \qquad \text{Equation 1}$$

We assume a normalized pure exponential decay model, that is, for an IR:

$$h(t) = \sqrt{\frac{\mathrm{RT}}{\mathrm{Ln}(10^6)}}\, e^{-\ln(1000)t/\mathrm{RT}} \qquad \text{Equation 2}$$

At a distance to the sound source r, with sample frequency Fs, the analytical expression of the convergence series in Equation 1 is:

$$CF_r(x) = 10 \cdot \mathrm{Log}_{10}\left(\frac{x \cdot \mathrm{Ln}(10^6)}{\mathrm{Fs} \cdot \mathrm{RT} \cdot \left(10^{\frac{6}{\mathrm{Fs} \cdot \mathrm{RT}}} - x\right)}\right) \qquad \text{Equation 3}$$

where *x* is a non-dimensional variable used to plot the CF.

By taking the Reverberation Time (RT) as a fundamental acoustic parameter of room design [2, 3], the theoretical CFs for various RT values are shown in Figure 1.

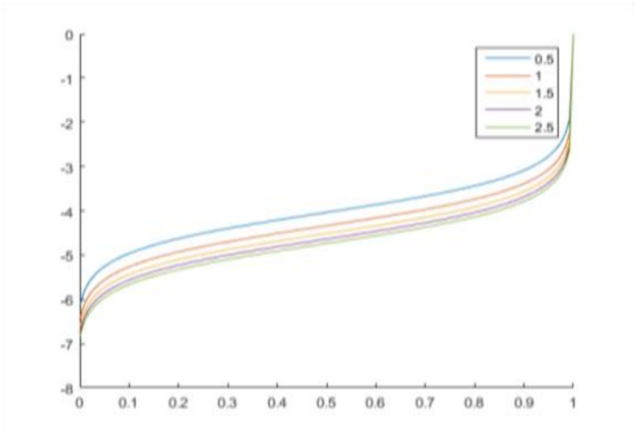


Figure 1. *CFs for a pure exponential decay model for various RT values (0.5 to 2.5 s)*

For its determination, the impulse response is measured by procedures clearly specified by the ISO 3882-1 standard [3]. However, if we want to ascertain only the RT parameter, then it suffices to limit ourselves to simply measuring the IR at several points with an omnidirectional microphone. Although RT presents no large variations in a room (around 3%), it is known that IRs have major variations in the same room in terms of their energy composition (to a greater or minor extent T_s, C_{50}, and C_{80} parameters give us an account of these variations in monaural IRs) [4, 5, 6].

The CF for the exponential model (Figure 1) should be almost constant in a room since it only depends on RT, and this parameter is practically constant (variations <3%). However, the CF obtained in the areas analysed show significant variability, as shown below.

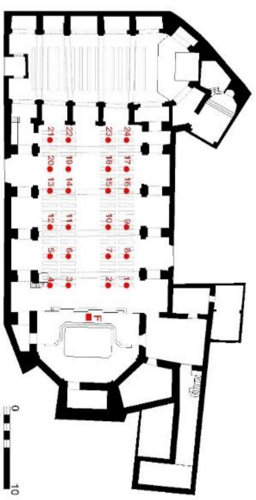
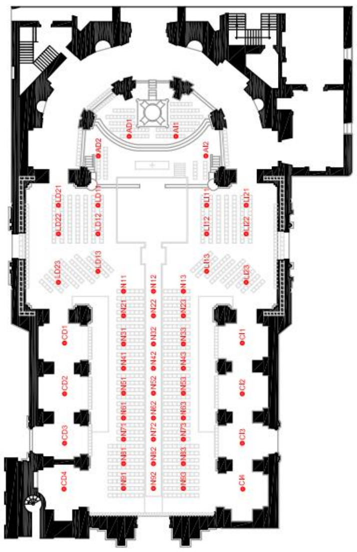
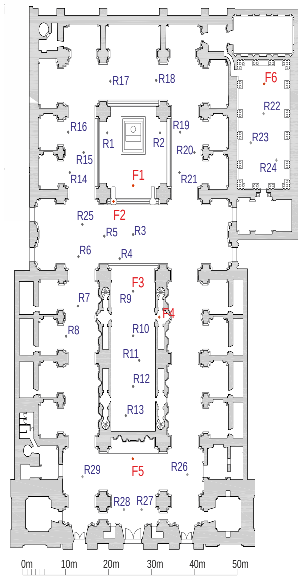
The study of the acoustics of places of worship has aroused major interest in recent decades [7]. On the one hand, this interest is of a practical nature as a result of the growing demand for acoustic comfort in public places that are used for oral or musical liturgical purposes or for other cultural performances [8, 9]. On the other hand, the interest is more fundamental, since the investigation into the acoustics of these complex spaces provides information regarding the general acoustic aspects of architectural heritage and aids in the general comprehension of room acoustics [10, 11]. In this work, we present the CFs of six worship buildings (cathedrals and basilicas). Their variation with respect to distance is analysed and the possible relationship of this variation with the usual energy parameters are studied: C50, C80, and G.

2. THE CHURCHES STUDIED

Table 1 shows the architectural and acoustics data of the rooms studied. Figure 2 displays the ground plans with the distribution of the points of measurement.

Table 1. *General data of the studied cathedrals.*

Enclosure	Architectural Style	Volume (m^3)	Description	Source Location	Number of Receivers	T_{30m} (s)
Jaen Cathedral	Renaissance (c. XVI - c. XVIII)	85,100	3 naves. 18 lateral chapels	High altar	17	8.05
Malaga Cathedral	Renaissance (c. XVI - c. XVIII)	118,500	3 naves. 15 lateral chapels	High altar	12	6.89
Toledo Cathedral	Gothic (c. XIII-c. XV)	125,000	5 naves. 15 lateral chapels	High altar	16	4.68
Valencia Cathedral	Gothic-Neoclassic (c. XIII-c. XVIII)	55,925	3 naves. 8 lateral chapels	High altar	47	4.65
Santa María de Elche Basilica	Baroque (c. XVII-c. XVIII)	22,600	1 nave. 8 lateral chapels	High altar	37	6.40
San Jaime Apóstol de Algemesi Basilica	Baroque (c. XVII-c. XVIII)	15,000	1 nave. 8 lateral chapels and 1 final chapel.	High altar	24	5.02

San Jaime Apóstol Basilica, Algemesi	Santa María Basilica, Elche	Jaen Cathedral
		

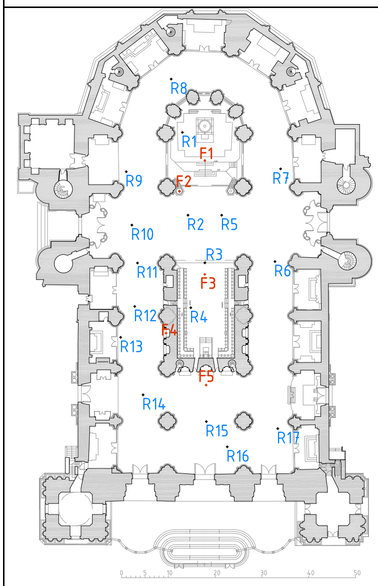
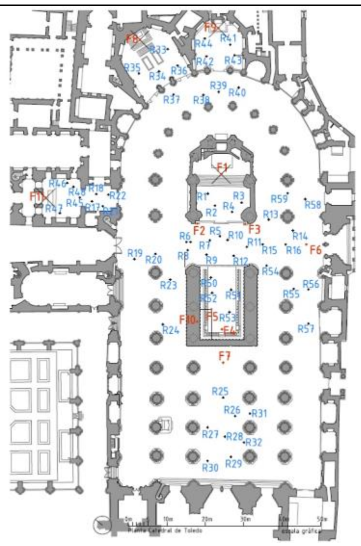
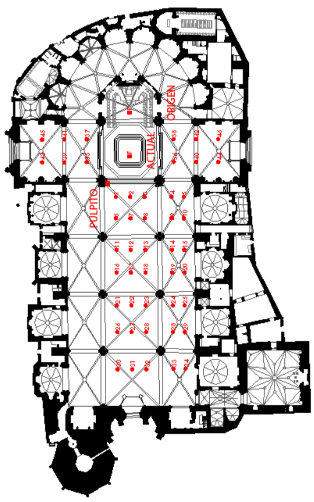
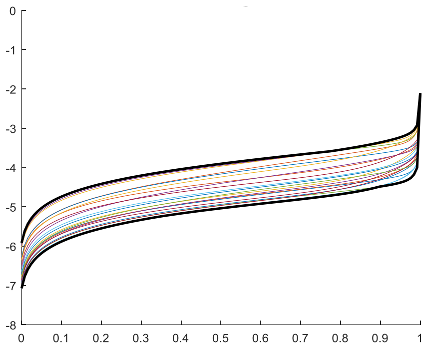
Malaga Cathedral	Toledo Cathedral	Valencia Cathedral
		

Figure 2. *Ground plans of the enclosures with the locations of the measuring points.*

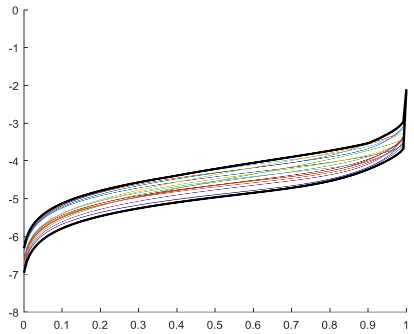
3. CHARACTERISTIC FUNCTION OF THE CHURCHES

Figure 3 shows the CF (in dB) calculated in the six places of worship for all the measuring positions with the upper and lower envelopes marked in black.

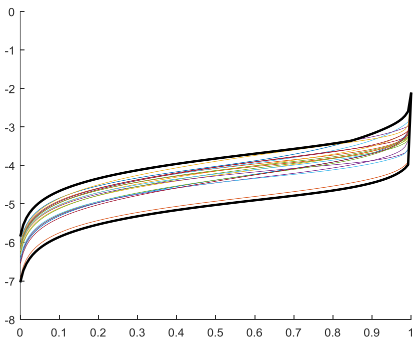
San Jaime Apóstol de Algemesi Basilica



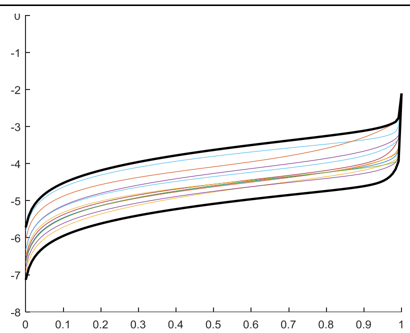
Santa María de Elche, Basilica



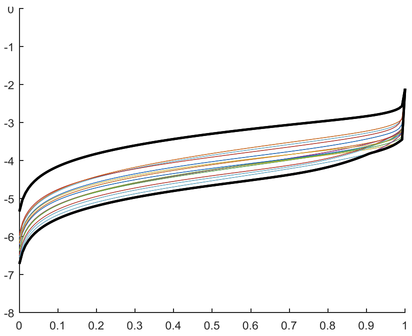
Jaen Cathedral



Malaga Cathedral



Toledo Cathedral



Valencia Cathedral

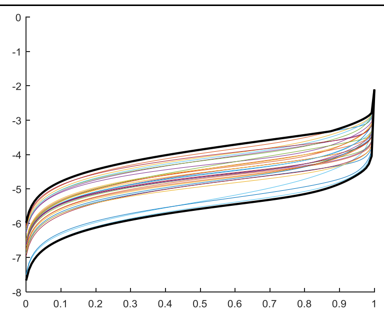


Figure 3. Characteristic Function (in dB) for the 6 worship places studied. The horizontal axis is the auxiliary non-dimensional variable x.

As a measure of the variation between two CFs, we calculate the difference in mean absolute point value. Although it can be seen in the figures that certain CFs are not parallel to each other and that crosslinks do occur, in this study we calculate the average difference with respect to the upper envelope line. We therefore obtain a MDcf parameter in dB, which, for the CFs next to the top line, is practically 0, while for the CFs next to the lower envelope, the highest value of positive MDcf is obtained.

Here we show the values of the MDcf (dB) parameter obtained as a function of source-receiver distance for each church (Figure 4).

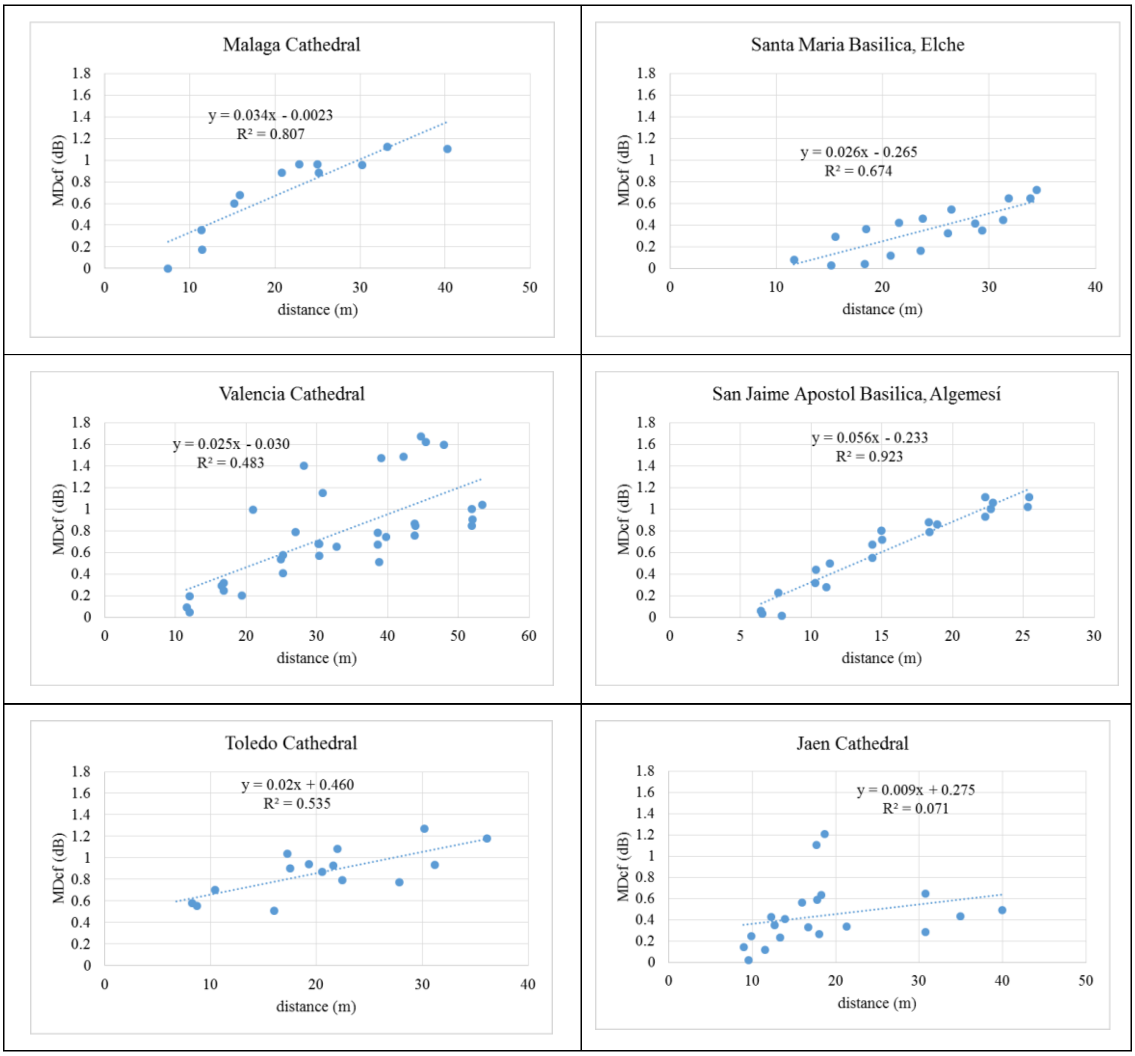


Figure 4. *Values of the MDcf parameter (mean difference of the CF corresponding to a distance to the source, with respect to the upper envelope line, in dB) as a function of the distance to the source. The linear adjustment and the correlation coefficient R^2 are represented.*

As can be observed, the dependence of the difference between the CFs as a function of distance is notable in some of the studied churches: The Basilica of Algemesi, the Basilica of Santa María de Elche and the Cathedral of Malaga. In the Cathedral of Valencia and the Cathedral of Toledo this dependence is somewhat lower, although their R2 is close to 50%. The only example in which the dependence on distance is very low corresponds to the Cathedral of Jaen. However, in this case, and also in almost all other buildings, there are 2 components in the grouping of points. By way of example, Figure 5 shows the corresponding adjustments of the cathedrals of Jaen and Valencia.

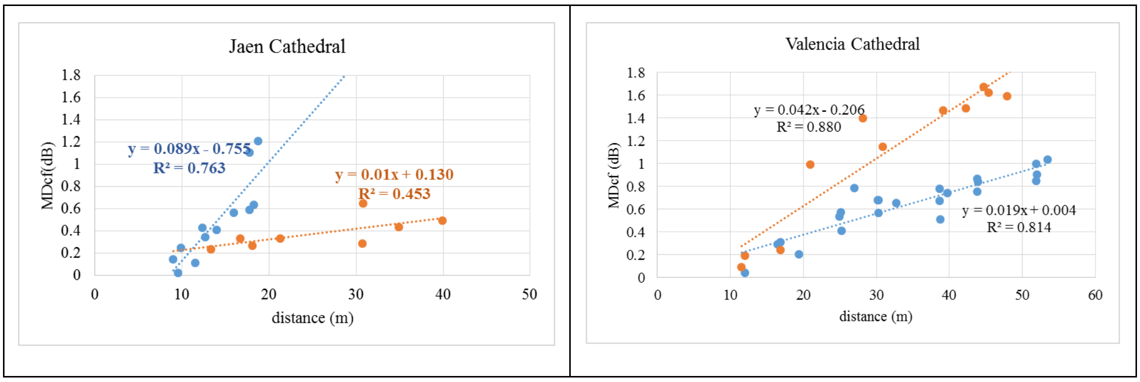


Figure 5. *Values of MDcf parameter (mean difference of the CF corresponding to a distance to the source, with respect to the upper envelope line) as a function of the distance to the source. The linear adjustments are represented considering two grouping factors.*

In general, it can be established in all enclosures analysed that there are two series that show a linear trend of the MDcf with distance (Table 2), with high values of R2 and with one of the components with a small variation with distances (≤ 3%) and another with a greater variation with distances that would be related to the energy parameters.

Table 2. *Slope, interception, and value of R^2, obtained by considering two components in the data obtained from the MDcf as a function of distance.*

	Slope	Interception	R^2 value
St. Jaime Apostol Basilica, Algemesi	0.067	-0.309	0.965
	0.058	-0.330	0.963
Malaga Cathedral	0.052	-0.19	0.979
	0.037	-0.178	0.902
Toledo Cathedral	0.031	0.346	0.881
	0.03	0.037	0.915
Jaen Cathedral	0.089	-0.755	0.763
	0.009	0.130	0.452
Santa Mª Basilica, Elche	0.024	-0.129	0.959
	0.029	-0.452	0.944
Valencia Cathedral	0.042	-0.206	0.880
	0.019	0.004	0.814

4. RELATIONSHIP WITH ENERGY PARAMETERS

Finally, we analyse the relationship between the parameter determined in the previous section for a single component and the energy parameters C_{50mid}, C_{80mid}, and G_{mid}. In three of the churches studied, a good correlation is observed between the MDcf parameter and the C_{50mid} and C_{80mid} values. Nevertheless, for the Basilica of Santa Maria de Elche and the Cathedral of Jaen, these correlations are very low, and in the case of the Cathedral of Valencia, correlation is practically non-existent, as shown in Figure 6. In this last case, however, the slopes of the 3 parameters are slightly less inclined than in the other cathedrals and basilicas.

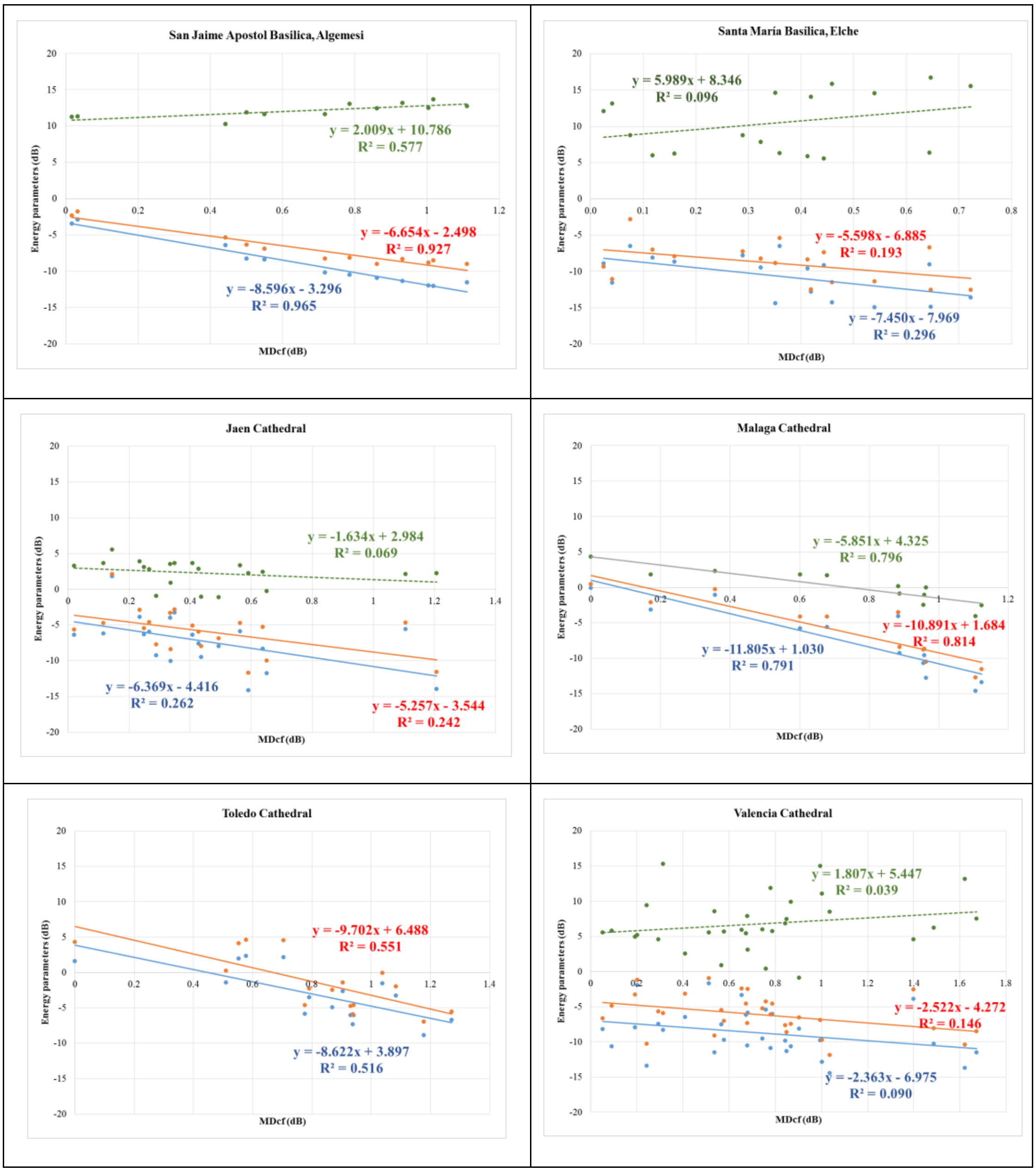


Figure 6. Relationship between the energy parameters (C_{50mid} in blue, C_{80mid} in red, and G_{mid} in green) as a function of MDcf value for each church and cathedral studied.

However, if these two buildings take into account the two series in which it appears that MDcf data are grouped as a function of distance, and the corresponding adjustments are made between $C_{50\text{mid}}$, $C_{80\text{mid}}$ and G_{mid}, then a significant increase in the correlations in all cases (Table 3) is observed, which becomes significant both for the Cathedral of Jaen and for the Basilica of Santa Maria de Elche. In the case of the Cathedral of Valencia, the correlation does not become significant and would need additional study.

The evaluation of the values of the slopes and interceptions that show the relationship between the energy parameters and MDcf values will be the subject of a later study which involves the analysis of a greater number of buildings.

Table 3. *Slope, interception, and value of R^2 corresponding to the relationship between the energy parameters (C_{50mid}, C_{80mid}, and G_{mid}) in dB, as a function of MDcf value in dB, for the two components that exist in the analysis of the MDcf versus distance.*

		First Component			Second Component		
	Energy parameter	Slope	Interception	R^2 value	Slope	Interception	R^2 value
Jaen Cathedral	C_{50mid}	-8.972	-4.646	0.379	-0.812	-2.366	0.360
	C_{80mid}	-7.932	-3.700	0.379	-6.865	-1.624	0.359
	G_{mid}	-8.409	4.107	0.500	-2.171	4.319	0.541
Santa Mª Basilica, Elche	C_{50mid}	0.741	-9.522	0.425	-13.122	-6.196	0.359
	C_{80mid}	4.077	-9.368	0.409	-13.529	-3.828	0.556
	G_{mid}	-8.873	10.28	0.424	16.249	5.583	0.442
Valencia Cathedral	C_{50mid}	-1.936	-7.059	0.164	-3.770	-6.248	0.088
	C_{80mid}	-2.168	-4.259	0.272	-3.844	-3.611	0.134
	G_{mid}	0.699	7.603	0.009	1.737	4.957	0.021

4. CONCLUSIONS

The Characteristic Function contains the IR information at each measuring point in a room and can be used for its characterization. The difference of the characteristic functions with source-receiver distance is related to the average energy parameters, although it would be necessary to study these energy parameters per frequency band.

It is necessary to obtain quantifying magnitudes of this characteristic function to enable the rooms studied to be qualified.

5. ACKNOWLEDGEMENTS

This work has been financially supported by the Spanish Ministry of Economy and Innovation within the research projects with references BIA2008-05485, BIA2012-36896, and BIA2016-76957-C3-3-R.

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